A DISCRETE-TIME MODEL FOR CODED DENSE WDM NETWORKS EMPLOYING FABRY-PEROT FILTERS

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Abstract

We present a novel discrete-time model to evaluate the capacity, power penalty/coding gain, and bit error probability of very dense WDM networks employing OOK modulation with direct detection and Fabry-Perot filter as channel selector. Reed-Solomon error-correcting codes and concatenated Reed-Solomon/Reed-Solomon codes are employed to pack the optical channels in the very dense network. The discrete-time model utilizes the trapezoidal expression together with signal samples obtained at the Nyquist rate equal to twice the Fabry-Perot spectral range.

1 Introduction

We investigate the capacity of very dense wavelength division multiplexing (WDM) networks in which the channels are packed extremely close to each other. Such networks can be realized by the use of powerful error-correcting codes suitable at very high bit rates. We propose the Reed-Solomon (RS) error-correcting codes and the concatenated Reed-Solomon/Reed-Solomon (RS/RS) codes. We propose these codes instead of the single error-correcting Hamming codes because very dense WDM networks encounter severe crosstalk degradation which renders Hamming codes virtually useless. Another reason for using RS/RS codes is that concatenated codes allow the use of two relatively short RS codes with modest error-correcting capability, one as the inner code and the other as the outer code to achieve almost the same coding gain as the equivalent long code. Short codes with modest error-correcting capability simplify decoder design at very high data rates. Also, Reed-Solomon codes allow the implementation of the coder and the decoder at the code symbol rate, which is much smaller than the bit rate; therefore, high-speed electronics can be used. These features make short RS codes and short concatenated RS/RS codes practical for WDM networks. Furthermore, the coding gain obtained by these codes provide a substantial increase in the number of optical channels in a very dense WDM network as compared to an uncoded WDM network. This is particularly attractive in an all optical local area network (LAN).

The analysis of very dense WDM networks poses a great challenge from the computational point of view. Error-correcting codes allow the optical channels to be spaced very closely. Analyses employing a frequency-domain approach using the Fourier transform [1–3] would result in prohibitive computer time. This happens because the beat interference between adjacent channels must be taken into account. Neglecting the beat interference as in the analysis of uncoded dense WDM networks reported in references [1–3] would not be appropriate for very dense WDM networks with error correcting codes. In this paper we present a novel and very computationally effective method called discrete-time analysis to assess the capacity of very dense coded WDM networks employing on-off keying (OOK) modulation with direct detection and a Fabry-Perot (FP) filter as the channel selector. The discrete-time analysis enables the evaluation of the bit error probability and power penalty with minimal computational effort.

When the very dense WDM network employs the Fabry-Perot (FP) filter as a channel selector, it is sufficient to use $N+1=\alpha T+1$ complex samples or $2(\alpha T+1)$ real samples (α is the free spectral range (FSR) of the FP filter and $1/T$ is the coded information rate) to evaluate the bit error probability and power penalty/coding gain including all intersymbol interference (ISI) and adjacent channel interference (ACI) effects. Assuming that the very dense WDM network operates with all channels confined to one FSR of the FP filter, then $\alpha T + 1$ complex signal samples during any coded bit interval of T seconds imply sampling the bit at a Nyquist rate of 2α (Hz). Coincidentally, the impulse response of the equivalent lowpass FP filter is an infinite sequence of impulse functions equally spaced in time by $1/\alpha$ seconds. Therefore, the FP filter practically provides $\alpha T + 1$ complex samples for the complex baseband (equivalent lowpass) very dense WDM signals. The $\alpha T + 1$ complex samples of a detected bit are sufficient to provide an accurate evaluation of the bit energy. In Section 2, the discrete-time coded analysis is presented. The numerical results are presented in Section 3 and the concluding remarks appear in Section 4.

2 Discrete-Time Analysis

The OOK receiver for the very dense WDM signal is shown in Fig. 1. The receiver contains a frequency selective FP filter to demultiplex one of the $M + 1$ channels. For convenience, we designate Channel 0 as the desired channel and Channel k as another channel where $k = -M/2, \cdots, -1, 1, \cdots, M/2$ and M is an even integer. The optical signal is received by the FP filter tuned to the wavelength of Channel 0, which allows the Channel 0 data signal to pass and rejects signals in the M adjacent channels. The photodetector demodulates the OOK signal to a baseband signal. We assume the photodetector has a responsivity $\mathcal{R}$. The baseband signal is then amplified by a low-noise amplifier which adds a postdetection thermal noise $\eta(t)$ with spectral density N_0. Both the signal plus noise are integrated over a coded bit interval T to obtain a decision variable Y, which is compared to a threshold V_T to determine whether an "0" or "1" was sent. The detected bits are then processed by the decoder to correct errors.

We consider the equivalent lowpass (complex envelope) data signals in Channel 0 and Channel k as follows:

$$b_0(t) = \sqrt{P} \sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) \tag{1}$$

$$b_k(t) = \sqrt{P} \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\phi_k} e^{j2\pi f_k t} p_T(t - \ell T) \tag{2}$$

where P is the received optical power per channel, $b_{0,i} \epsilon \{0, 1\}$ is the bit in Channel 0 during the i-th time period $(iT, (i+1)T)$, L_0 is the number of bits which trail the detected bit $b_{0,0}$, $b_{k,\ell} \epsilon \{0, 1\}$ is the bit in Channel k during the ℓ-th time period $(\ell T, (\ell + 1)T)$, L is the number bits in Channel k which trail the bit $b_{k,0}$, ϕ_k is the Channel k phase offset from Channel 0 (assumed to be uniformly distributed in $(0, 2\pi)$ radians), f_k is the frequency spacing between Channel k and Channel 0 and $f_k = -f_{-k}$. The pulse function $p_T(t)$ is defined as

$$p_T(t - iT) = \begin{cases} 1, & iT < t \leq (i+1)T \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

The received equivalent lowpass optical signal at the input of the FP filter is given by

$$\begin{aligned} r_i(t) &= \sqrt{P} \sum_{i=-L_0}^{0} b_{0,i} p_T(t - iT) \\ &\quad + \sqrt{P} \sum_{k=-M/2}^{M/2} \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\phi_k} e^{j2\pi f_k t} p_T(t - \ell T) \quad k \neq 0. \end{aligned} \tag{4}$$

The ideal single-cavity FP filter is a causal, linear, time-invariant filter whose equivalent lowpass impulse response is given as follows [1]:

$$h(t) = (1 - r) \sum_{m=0}^{\infty} r^m \delta\left(t - \frac{m}{\alpha}\right) \tag{5}$$

where r is the power reflectivity and α is the free spectral range. We assume that α is a multiple of the coded bit rate $1/T$, that is, αT is a positive integer.

Performing the convolution of (4) and (5) and restricting the output $s(t)$ of the FP filter to the detected interval $0 \leq t \leq T$, we obtain

$$s(t) = s_D(t) + s_{ISI}(t) + s_{ACI}(t) \tag{6}$$

where $s_D(t)$ is the filtered desired signal, $s_{ISI}(t)$ is the filtered ISI signal, and $s_{ACI}(t)$ is the filtered ACI signal. These three terms are evaluated as follows:

$$\begin{aligned} s_D(t) &= \sqrt{P}\, b_{0,0} p_T(t) * (1-r) \sum_{m=0}^{\infty} r^m \delta\left(t - \frac{m}{\alpha}\right) \\ &= (1-r)\sqrt{P} b_{0,0} \sum_{m=0}^{[\alpha t]} r^m p_T\left(t - \frac{m}{\alpha}\right), \quad 0 \leq t \leq T \end{aligned} \tag{7}$$

where the notation * stands for convolution operation and $[\alpha t]$ is the integer part of αt. The ISI term is

$$\begin{aligned} s_{ISI}(t) &= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} p_T(t - iT) * (1-r) \sum_{m=0}^{\infty} r^m \delta\left(t - \frac{m}{\alpha}\right) \\ &= (1-r)\sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \sum_{m=[\alpha t]-(i+1)\alpha T}^{[\alpha t]-i\alpha T} r^m p_T\left(t - iT - \frac{m}{\alpha}\right), \\ &\qquad 0 \leq t \leq T\,. \end{aligned} \tag{8}$$

Similarily, using the above procedure in (7) and (8) the ACI term can be evaluated as

$$\begin{aligned} s_{ACI}(t) &= \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\ell=-L}^{0} b_{k,\ell} e^{j\phi_k} e^{j2\pi f_k t} p_T(t - \ell T) * (1-r) \sum_{m=0}^{\infty} r^m \delta\left(t - \frac{m}{\alpha}\right) \\ &= (1-r)\sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} b_{k,0} e^{j\phi_k} \sum_{m=0}^{[\alpha t]} r^m e^{j2\pi f_k (t - m/\alpha)} p_T\left(t - \frac{m}{\alpha}\right) \\ &\quad + (1-r)\sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{\ell=-L}^{-1} b_{k,\ell} e^{j\phi_k} \sum_{m=[\alpha t]-(\ell+1)\alpha T}^{[\alpha t]-\ell\alpha T} r^m e^{j2\pi f_k (t - m/\alpha)} p_T\left(t - \ell T - \frac{m}{\alpha}\right), \\ &\qquad 0 \leq t \leq T\,. \end{aligned} \tag{9}$$

The decision variable Y appears at the output of the integrator. It consists of a signal component X and an amplifier noise component N,

$$Y = X + N_A\,, \tag{10}$$

where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \tag{11}$$

and

$$N_A = \int_0^T \eta(t) dt\,. \tag{12}$$

Here $\eta(t)$ is a zero-mean Gaussian process of spectral density N_0. Thus, N_A is a Gaussian random variable with zero mean and variance N_0T.

Let $Z = \mathcal{R}P\sqrt{T_b/N_0} = \mathcal{R}P\sqrt{T/r_cN_0}$ be the signal-to-noise ratio, where $T_b = T/r_c$ is the uncoded bit duration and r_c is the code rate. We define the power penalty/coding gain (dB) as the increase/decrease in Z (dB) required for the WDM network to achieve a required bit error probability P_e at the output of the decoder over the uncoded single channel operation without any filtering effect.

For a detection threshold V_T and an ISI/ACI bit pattern $b = \{b_{0,i}, b_{k,\ell}e^{j\phi_k}\}$ for $k = -M/2, \cdots, M/2$, $k \neq 0$ and $\ell = -L, \cdots, 0$, and $i = -L_0, \cdots, -1$; the conditional transition probability error for Channel 0 is given by

$$P_c(b) = \frac{1}{2}Q\left(\frac{X_1(b) - V_T}{\sqrt{N_0T}}\right) + \frac{1}{2}Q\left(\frac{V_T - X_0(b)}{\sqrt{N_0T}}\right) \tag{13}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy \tag{14}$$

and $X_0 = X$ when $b_{0,0} = 0$ and $X_1 = X$ when $b_{0,0} = 1$.

We consider concatenated RS/RS with the inner code (n_1, k_1) and outer code (n_2, k_2). The inner code has length $n_1 = 2^{m_1} - 1$ with m_1 bits per symbol and can correct $t_1 = (n_1 - k_1)/2$ symbol errors. The outer code is a shortened RS code [4] with length n_2 and error correcting capability $t_2 = (n_2 - k_2)/2$. The RS/RS code rate is $r_c = k_1k_2/n_1n_2$.

For a given (n, k) RS code with $t = (n - k)/2$ error correcting capability, the symbol error probability p_s at the decoder output is given by [4–5]

$$p_s < \sum_{j=t+1}^{n} \frac{j+t}{n}\binom{n}{j}p^j(1-p)^{n-j} \tag{15}$$

where p is the symbol error probability at the RS decoder input. For the concatenated RS/RS code the symbol error probability p_{s_1} at the output of the inner RS decoder is determined by (15) with $p = 1 - (1 - P_c)^{m_1}$ where P_c is the channel transition probability given in (13). The symbol error probability p_{s_2} at the output of the outer RS decoder is determined by (15) with p replaced by p_{s_1}. The bit error probability P_e is obtained by multiplying p_{s_2} with the factor $(n_2 + 1)/2n_2$ to account for the average number of information errors per symbol error [5]. This P_e is then averaged out over all ISI/ACI bit patterns.

The evaluation of (11) and subsequently (13) poses a great challenge for very dense WDM networks. A straight-foward computation of X in (11) using (6)–(9) would be computationally prohibitive. One approximation would be to neglect the beat interference term $|s_{ACI}(t)|^2$ as done previously in [1–3]. For a very dense WDM network this would result in an overestimated capacity. In the following discussion we introduce a discrete-time analysis to evaluate (11) and (13) without neglecting any terms in $|s(t)|^2$ and with highly efficient computation.

The discrete-time analysis employs $N + 1 = \alpha T + 1$ complex samples of $s(t)$ in (6)–(9) to evaluate the integration of $|s(t)|^2$ in (11) with a trapezoidal expression,

$$X = \frac{\mathcal{R}}{2\alpha}\left\{|s(t_0)|^2 + |s(t_N)|^2 + 2\sum_{n=1}^{N-1}|s(t_n)|^2\right\} . \tag{16}$$

In practice, the values of the spectral range-coded bit duration $N = \alpha T$ range from a few hundreds to a few thousands, thus the right side of (16) can be evaluated with a very high degree of accuracy.

We now take samples of $s(t)$ at time $t_n = n/\alpha$, $n = 0, 1, \cdots, N = \alpha T$. There are $N + 1 = \alpha T + 1$ complex samples, or equivalently $N + 1$ samples for the in-phase component of $s(t)$ and $N + 1$ samples for the quadrature component of $s(t)$. Thus, we sample the signal $s(t)$ at the Nyquist rate assuming that the very dense WDM signal $s(t)$ is totally confined to the spectral range α of the FP filter. (That is, the absolute bandwidth of $s(t)$ is α.) Using (6) we obtain

$$s(t_n) = s\left(\frac{n}{\alpha}\right) = s_D(t_n) + s_{ISI}(t_n) + s_{ACI}(t_n) . \tag{17}$$

The terms $s_D(t_n)$, $s_{ISI}(t_n)$, and $s_{ACI}(t_n)$ can be evaluated from (7), (8), and (9) as follows:

$$\begin{aligned} s_D(t_n) = s_D\left(\frac{n}{\alpha}\right) &= (1-r)\sqrt{P}\,b_{0,0}\sum_{m=0}^{[\alpha t_n]} r^m p_T\left(t_n - \frac{m}{\alpha}\right) \\ &= \sqrt{P}\,b_{0,0}(1-r^n), \qquad 0 \le n \le N . \end{aligned} \tag{18}$$

We have used the fact that $p_T[(n-m)/\alpha] = 1$ if $m < n$ and $p_T[(n-m)/\alpha] = 0$ if $m = n$, by definition (3). For the term $s_{ISI}(t_n)$ we have

$$\begin{aligned} s_{ISI}(t_n) = s_{ISI}\left(\frac{n}{\alpha}\right) &= (1-r)\sqrt{P}\sum_{i=-L_0}^{-1} b_{0,i} \sum_{m=[\alpha t_n]-(i+1)N}^{[\alpha t_n]-iN} r^m p_T\left(t_n - iT - \frac{m}{\alpha}\right) \\ &= r^n(r^{-N}-1)\sqrt{P}\sum_{i=-L_0}^{-1} b_{0,i}r^{-iN}, \qquad 0 \le n \le N . \end{aligned} \tag{19}$$

Similarly, the term $s_{ACI}(t_n)$ can be evaluated as

$$\begin{aligned} s_{ACI}(t_n) &= (1-r)\sqrt{P}\sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} b_{k,0}e^{j\phi_k}\sum_{m=0}^{[\alpha t_n]} r^m e^{2\pi f_k(t_n - m/\alpha)} p_T\left(t_n - \frac{m}{\alpha}\right) \\ &\quad +(1-r)\sqrt{P}\sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2}\sum_{\ell=-L}^{-1} b_{k,\ell}e^{j\phi_k}\sum_{m=[\alpha t_n]-(\ell+1)\alpha T}^{[\alpha t_n]} r^m e^{2\pi f_k(t_n - m/\alpha)} \\ &\quad \cdot p_T\left(t_n - \ell T - \frac{m}{\alpha}\right) \\ &= (1-r)\sqrt{P}\sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} b_{k,0}e^{j\phi_k}\left[\frac{e^{j2\pi n f_k/\alpha} - r^n}{1 - re^{-j2\pi f_k/\alpha}}\right] \\ &\quad \cdot(1-r)\sqrt{P}\sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2}\sum_{\ell=-L}^{-1} b_{k,\ell}e^{j\phi_k}e^{j2\pi n f_k/\alpha} \\ &\quad \cdot\left[\frac{(re^{-j2\pi f_k/\alpha})^{n-(\ell+1)N} - (re^{-j2\pi f_k/\alpha})^{n-\ell N}}{1 - re^{-j2\pi f_k/\alpha}}\right], \qquad 0 \le n \le N . \end{aligned} \tag{20}$$

The results in (17)–(20) can be used in (11), (13)–(14) to obtain the power penalty/coding gain and the average bit error probability with the following threshold

$$V_T = \frac{1}{2}\left(X_{1,\min} + X_{0,\max}\right) \tag{21}$$

where $X_{1,\min} = \min\{X\}$ when $b_{0,0} = 1$ and $X_{0,\max} = \max\{X\}$ when $b_{0,0} = 0$ with X given in (16).

3 Numerical Results

In this section we present the numerical results obtained via the discrete-time analysis for both concatenated RS/RS codes (15, 13)/(13, 11) and RS code (31, 27). The FP filter has a spectral range $\alpha = 3800$ GHz, a power reflectivity $r = 0.99$ and hence the finesse is $\pi\sqrt{r}/(1-r) = 312.6$. The number of samples taken is $N = \alpha T = 2000$. *Thus the corresponding coded bit rate per channel is* $1/T = 1.9$ *Gb/s*. The total number of channels in the coded network is $M + 1 = \alpha T/I$ where I is the channel separation normalized to $1/T$. The aggregate network capacity C is the number of channels times the bit rate $1/T_b = r_c/T$ where r_c is the code rate, that is, $C = (M + 1)r_c/T = r_c\alpha/I$. Figure 2 shows the power penalty/coding gain (dB) versus the normalized channel separation I for the (15, 13)/(13, 11) codes at a bit error probability of 10^{-15}. For a power penalty of 1.2 dB, the resulting normalized channel separation is $I = 8$ yielding an aggregate network capacity $C = (11/15)(3800)/8 = 348.3$ Gb/s. Also plotted in Fig. 2 is the power penalty for an uncoded network. We use the same number of samples, that is, $N = \alpha T_b = 2000$. The channel separation is now normalized to the bit rate $1/T_b$. The total number of channels is $M + 1 = \alpha T_b/I$ and the aggregate network capacity is $C = (M+1)/T_b = \alpha/I$. For the same power penalty of 1.2 dB, the resulting normalized channel spacing is 14 yielding an aggregate network capacity $C = (3800)/14 = 271.4$ Gb/s as compared to 348.3 GB/s for a coded network. Thus the use of (15, 13)/(13, 11) codes provides approximately 28% increase in capacity. On the other hand, assume that the aggregate capacity of the coded network is the same as that of the uncoded network, that is, 271.4 Gb/s. Then, the channel separation normalized to the coded bit rate $1/T$ is 10.3. This corresponds to a coding gain of 0.2 dB. The aggregate power gain versus the uncoded network is therefore equal to 1.4 dB.

In Fig. 3, the performance of the coded network is shown as bit error probability versus the input signal-to-noise ratio with the normalized channel separation as a parameter. Also plotted is the curve for a single channel operation without any filtering effect. In Fig. 4, the power penalty/coding gain (dB) versus normalized channel separation is shown for RS code (31, 27). This code has an error correcting capability of two errors as compared to one error for both inner and outer codes of the concatenated codes (15, 13)/(13, 11). At 1.2 dB power penalty the channel separation is about 8.4 yielding an aggregate network capacity $C = (27/31)(3800)/8.4 = 394$ Gb/s. Thus the use of code (31, 27) provides an approximate 45% increase in capacity when compared to the capacity of an uncoded network. If the aggregate capacity of the (31, 27) coded network is set to be the same as that of the uncoded network (271.4 Gb/s), then the normalized channel separation is 12.21. This yields a coding gain of 0.5 dB and hence an aggregate power gain versus the uncoded network of 1.7 dB.

In general, a single RS code performs slightly better than concatenated RS/RS codes with the same error correcting capability. The advantage of the concatenated codes is that the inner and outer codes need to have only half the error correcting capability, making them much easier to implement at high bit rates.

4 Conclusion

We have presented a novel discrete-time model to analyze the bit error probability and power penalty and coding gain of direct detection WDM networks employing OOK modulation and single cavity FP filter at the receiver. The discrete-time approach enables the decision variable to be evaluated by the trapezoidal expression based on $\alpha T + 1$ complex samples of the complex envelope of the WDM signal at the output of the FP filter. The simplicity of the model makes the computation highly efficient. Since the number of samples $\alpha T + 1$ is usually large for practical FP filters, the results are highly accurate. This is especially true for coded networks with closely packed channels where the beat interference between ACI channels cannot be ignored and the calculation is too computationally prohibitive to be included as reported in [1].

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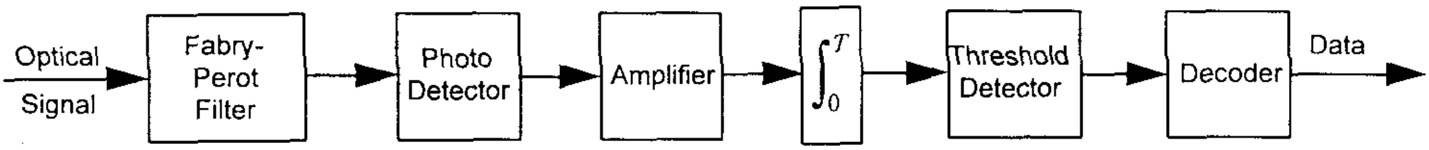

Figure 1: OOK receiver for very dense WDM signal with error-correcting codes.

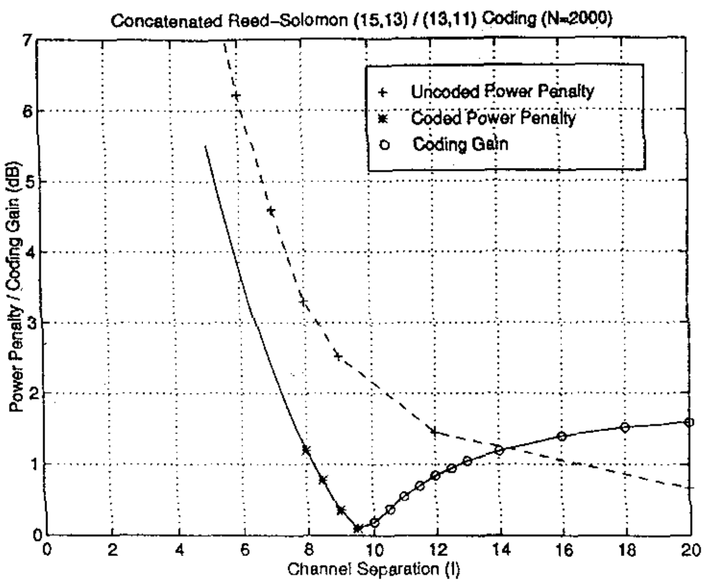

Figure 2: Power penalty/coding gain versus normalized channel separation for concatenated (15, 13)/(13, 11) codes with $N = 2000$. The channel separation is normalized to the coded bit rate $1/T$ for the coded network, and to the bit rate $1/T_b$ for the uncoded network.

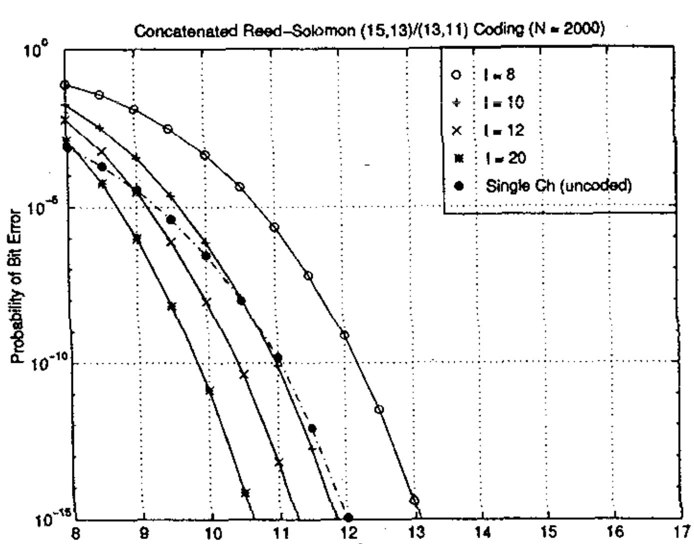

Figure 3: Bit error probability versus signal-to-noise ratio as a function of normalized channel separation for $N = 2000$.

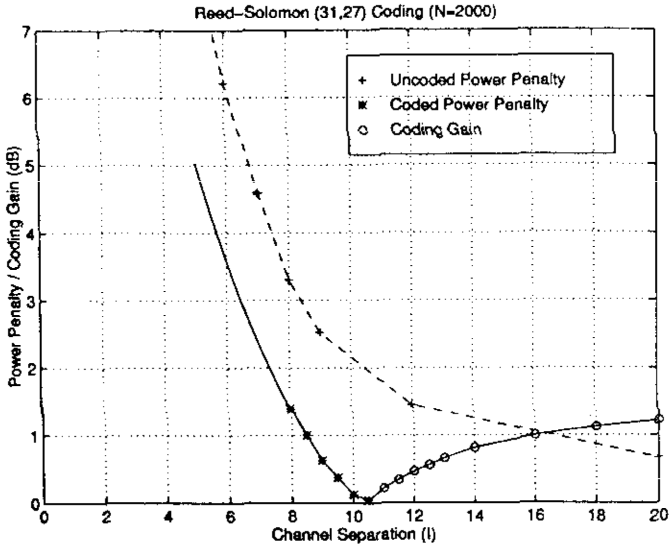

Figure 4: Power penalty/coding gain versus normalized channel separation for (31,27) code with $N = 2000$. The channel separation is normalized to the coded bit rate $1/T$ for the coded network, and to the bit rate $1/T_b$ for the uncoded network.

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